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Axial Currents from CKM Matrix CP Violation *

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Abstract

The first principle derivation of kinetic transport equations suggests that a CP-violating mass term during the electroweak phase transition can induce axial vector currents. Since the important terms are of first order in gradients there is a possibility to construct new rephasing invariants that are proportional to the CP phase in the Cabibbo-Kobayashi-Maskawa matrix and to circumvent the upper bound of CP-violating contributions in the Standard Model, the Jarlskog invariant.

I. INTRODUCTION

All models that intend to describe the baryon asymmetry of the universe (BAU) by electroweak baryogenesis (EWB)[1] depend on extensions of the Standard Model (SM) since the SM fails on the following grounds:

A) First order phase transition: Sakharov[2] pointed out that baryogenesis necessarily requires non-equilibrium physics. The expansion of the universe is too slow at the electroweak scale and one needs bubble nucleation during a first order EWPT. The phase diagram of the Standard Model is studied in detail[3], and it is well known that there is no first order phase transition in the Standard Model for the experimentally allowed Higgs mass.

B) Sphaleron bound: To avoid washout after the phase transition, the *vev* of the broken Higgs field has to meet the criterion $\langle \Phi \rangle \gtrsim T_c$, i.e. a strong first order phase transition. This

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results in the Shaposhnikov bound on the Higgs mass[4].

C) Lack of CP violation: Since the only source of CP violation in the Standard Model is the Cabibbo-Kobayashi-Maskawa (CKM) matrix (apart from the neutrino mass matrix, which provides an even tinier source of CP violation) one has to face that it is too weak to account for the observed magnitude of BAU.

In the following we will address the last point, the lack of sufficient CP violation, in the framework of kinetic theory.

The strong first order phase transition is assumed to occur at about $T_c \simeq 100$ GeV and is parametrised by the velocity of the phase boundary (wall velocity) v_w and its thickness l_w.

In this article we will focus on the following main points:

- We will demonstrate how CP violating sources can arise in semiclassical Boltzmann type equations.
- We argue that the Jarlskog determinant as an upper bound of CP violation in the SM is not valid during the EWPT.
- We give a rough estimate of the CP violating sources during the EWPT and conclude that the source is by orders larger than considering the Jarlskog determinant but still insufficient to explain the magnitude of the BAU.

II. AXIAL CURRENTS IN KINETIC THEORY

Starting point in kinetic theory are the exact Schwinger-Dyson equation for the two point functions in the closed time-path (CTP) formalism.

$$\begin{aligned} & e^{-i\Diamond}\{S_0^{-1} - \Sigma_R, S^<\} - e^{-i\Diamond}\{\Sigma^<, S_R\} \\ & \quad = \frac{1}{2} e^{-i\Diamond}\{\Sigma^<, S^>\} - \frac{1}{2} e^{-i\Diamond}\{\Sigma^>, S^<\}, \\ & e^{-i\Diamond}\{S_0^{-1} - \Sigma_R, \mathcal{A}\} - e^{-i\Diamond}\{\Sigma_A, S_R\} = 0, \end{aligned} \tag{1}$$

where we have used the definitions and relations

$$\begin{aligned} & S^{\bar{t}} := S^{--},\ S^t := S^{++},\ S^< := S^{+-},\ S^> := S^{-+}, \\ & \mathcal{A} := \frac{i}{2}(S^< - S^>), S_R := S^t - \frac{i}{2}(S^< + S^>), \\ & 2\Diamond\{A, B\} := \partial_{X^\mu} A \partial_{k_\mu} B - \partial_{k_\mu} A \partial_{X^\mu} B. \end{aligned} \tag{2}$$

S denotes the Wightman function, S_0^{-1} the free inverse propagator and Σ the selfenergy. $S^{\pm\pm}$ denotes the entries of the 2×2 Keldysh matrices and all functions depend on the average coordinate X and the momentum k. To simplify the equations, one can perform a gradient expansion. The terms on the left hand side will be expanded up to first order, whereas the collision terms on the right hand side vanish in equilibrium and are just kept up to zeroth order. The expansion parameter is formally ∂_X/k, which close to equilibrium and for typical thermal excitations reduces to $(l_w * T_c)^{-1}$.

We will not solve the full transport equations, but only look for the appearance of CP-violating source terms.

To start with, we consider a toy model with only one flavour and a mass term that contains a space dependent complex phase[5]. The inverse propagator in a convenient coordinate system reads

$$k_0\gamma_0 + k_3\gamma_3 + m_R(X_3) + im_I(X_3)\gamma_5. \tag{3}$$

Using the spin projection operator $P_s = \frac{1}{2}(1 + s\gamma_0\gamma_3\gamma_5)$ the Schwinger-Dyson equations can be decoupled and finally yield ($me^{i\theta} = m_R + i\, m_I$)

$$\left(k_0^2 - k_3^2 - m^2 + s\frac{m^2\theta'}{k_0}\right) Tr(\gamma_0 S_s^<) \;=\; 0 \tag{4}$$

$$\left(k_3\partial_{X_3} - \frac{(m^2)'}{2}\partial_{k_3} + s\frac{(m^2\theta')'}{2k_0}\partial_{k_3}\right) Tr(\gamma_0 S_s^<) \;=\; Coll. \tag{5}$$

We see, that in our approximation, the quasi-particle picture is still valid, since the Wightman function fulfills an algebraic constraint. Furthermore, the first order corrections lead to some source term, that is proportional to the complex phase of the mass and therefore CP violating.

Performing the calculation with several flavours, one finds the generalization of this CP violating term, reading $Tr(m^{\dagger\prime} m - m^\dagger m')$.

III. ENHANCEMENT OF CP VIOLATION IN THE SM

Jarlskog proposed an upper bound for CP violating effects in the Standard Model. Following her argument of rephasing invariants, the first CP violating quantity constructed out

of the Yukawa couplings is the Jarlskog determinant[7]

$$\begin{aligned} & Im\, det[\tilde{m}_u\, \tilde{m}_u^\dagger, \tilde{m}_d\, \tilde{m}_d^\dagger] \\ &= Tr(Cm_u^4 C^\dagger m_d^4 C m_u^2 C^\dagger \tilde{m}_d^2) \\ &\approx -2J \cdot m_t^4 m_b^4 m_c^2 m_s^2, \end{aligned} \tag{6}$$

When applied to the case of electroweak baryogenesis, one finds the upper bound of the BAU[8]

$$[\tfrac{g_W^2}{2M_W^2}]^7 J \cdot m_t^6 m_b^4 m_c^2 m_s^2 \approx 10^{-22}, \tag{7}$$

Though, two assumptions, that are needed for this argument are not fulfilled during the electroweak phase transition[6].

A) Since the mass matrix is space dependent, one needs space dependent diagonalization matrices to transform to the mass eigenbasis. This leads to new physical relevant quantities, that can as well be CP violating. As a generalization of the CP violating source term in the kinetic toy model above, we found $Tr(m^{\dagger\prime} m - m^\dagger m')$. However in the Standard model this term vanishes at tree level, for the mass matrix is proportional to its derivative.

B) The argument of Jarlskog is based on the fact, that the examined quantity is perturbative in the Yukawa coupling. The calculation of the selfenergy in a thermal plasma involves integrations over divergent logarithms, of the form

$$\begin{aligned} h_2(\omega,\kappa) &= \frac{1}{\kappa}\int_0^\infty \frac{d|\mathbf{p}|}{2\pi}\Big(\frac{|\mathbf{p}|}{\epsilon_h} L_2(\epsilon_h,|\mathbf{p}|) f_B(\epsilon_h) \\ &\qquad - \frac{|\mathbf{p}|}{\epsilon_u} L_1(\epsilon_u,|\mathbf{p}|) f_F(\epsilon_u)\Big). \end{aligned}$$

$$\begin{aligned} L_{1/2}(\epsilon,|\mathbf{p}|) &= \log\left(\frac{\omega^2-\kappa^2\pm\Delta+2\epsilon\omega+2\kappa|\mathbf{p}|}{\omega^2-\kappa^2\pm\Delta+2\epsilon\omega-2\kappa|\mathbf{p}|}\right) \\ &\quad +\log\left(\frac{\omega^2-\kappa^2\pm\Delta-2\epsilon\omega+2\kappa|\mathbf{p}|}{\omega^2-\kappa^2\pm\Delta-2\epsilon\omega-2\kappa|\mathbf{p}|}\right), \end{aligned}$$

that lead to a significant space dependence of the selfenergy. Since the space dependence is due to a resonance with the plasma particles, the selfenergy is highly sensitive to the quark masses and the W mass, that both change continously in the wall profile. However since CP violating effects only appear as an interference of the two loop and the one loop term, an estimation of the source term leads to the upper bound[6]

$$\frac{\delta\omega}{\omega} \sim J\cdot m_t^4 m_s^2 m_b^2 m_c^2 \frac{\alpha_w^3 h_2'}{m_W^8 l_w T^3} \approx 10^{-15}.$$

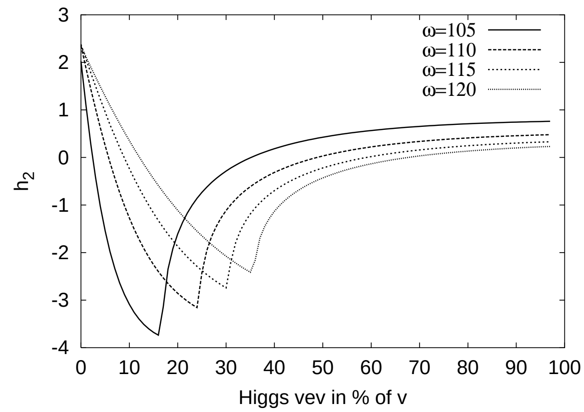

FIG. 1: Dependence of h_2 on the Higgs vev Φ in % of its value $\Phi^0 = 246$ GeV at T=0. The external energies and momenta are fixed at $\omega = 105$ GeV to $\omega = 120$ GeV, k=100 GeV, the mass of the quark in the loop is $m_u = 100$ GeV.

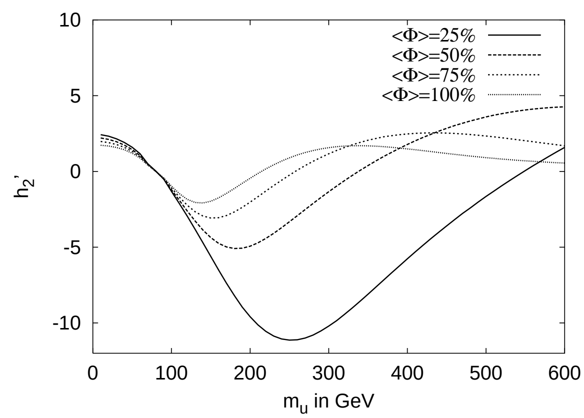

FIG. 2: Dependence of h_2' on the mass of the quark in the loop with an on-shell external quark of mass $m_e = 4$ GeV. The Higgs vev is chosen in a range of 25% to 100% of its value in the broken phase at T=0.

We conclude, that the axial current is enhanced seven orders in magnitude. Still the CP-violating source due to the CKM matrix might be too weak to account for the BAU.

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